Note that the solution of all the problems elucidated is based on the solution of the selfsimilar problem regarding a sudden change in the deformation on the boundary of an elastic half-space. It was found in the solution of this problem /8, 9/ that for certain relationships between the anisotropy and the initial deformations, a domain of values u_α^*, although small, can appear for which the solution is not unique. For these values of u_α^* additional investigations are necessary. Such an investigation is performed in /11/ and enables one to say to which of the two possible solutions preference should be given.

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CONSTRUCTION OF DISCONTINUOUS SOLUTIONS OF THE EQUATIONS OF PLANE ELASTICITY THEORY BY THE METHOD OF GENERALIZED FUNCTIONS*

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A method of constructing integral representations of discontinuous solutions of the equations of plane elasticity theory based on the use of the apparatus of the theory of generalized functions is described. The representations obtained for the discontinuous displacement and stress field components are utilized to formulate sufficient conditions ensuring continuous continuation of these quantities at almost all the points of the line of discontinuity.

1. Formulation of the problem. We consider the complete system of equations of plane elasticity theory describing the state of plane strain of a cylindrical body when there are no mass forces and initial stresses /1/ in a system of rectangular Cartesian coordinates

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x_1x_2

$$\partial\sigma_{ij}/\partial x_j = 0 \tag{1.1}$$

$$\sigma_{ij} = \lambda\frac{\partial u_k}{\partial x_k}\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1.2}$$

Here σ_{ij} and u_i are the components of the stress tensor and displacement vector in the coordinate system x_1x_2, λ, μ are the elastic Lame constants, and δ_{ij} is the Kronecker delta. In (1.1) and (1.2) and the subsequent formulas the subscripts i, j, k, l, m take the values 1, 2 and summation is over the repeated subscripts.

Let L be a rectifiable piecewise-smooth finite line represented parametrically: $L = \{(x_1, x_2): x_i = \xi_i(s),\ a \leqslant s \leqslant b\}$ in the x_1x_2 coordinate system. Here s is the arc abscissa measured from a certain fixed point on the contour L or on its continuation, a, b are real numbers whose difference determines the length of the line L, and $\xi_i(s)$ are bounded functions having piecewise-continuous derivatives in the interval (a, b). We assume that the set of points of the line L is closed.

The points of the coordinate plane x_1x_2 belonging to the contour L and corresponding to a define value of the parameter s will be denoted by $t(s) = (\xi_1(s), \xi_2(s))$; we indicate the remaining points of the plane by the symbols $x = (x_1, x_2)$ and $y = (y_1, y_2)$. The distance between the points of the coordinate plane is given by the expression $|x - y| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$.

We shall say that the point $t \in L$ has a multiplicity n for a given method of parametrizing the contour if n different values of the arc abscissa $s_1, s_2, \ldots, s_n$ exist for which $t(s_1) = t(s_2) = \ldots = t(s_n) = t$.

Obviously all points of selfintersection of the contour L (if such there be) have a multiplicity greater than one.

We consider the direction of the unit vector normal to the contour L to be positive if the normal is on the right during traversal of the contour in the direction of increase of the parameter s. If $\varphi_k(s)$ are angles formed by the direction of the positive normal at the point $t(s)$ and the coordinate axes x_k, then

$$\cos\varphi_1 = d\xi_2/ds, \quad \cos\varphi_2 = -d\xi_1/ds \tag{1.3}$$

We will call the set of functions σ_{ij}, u_i that are analytic in the open domain $R^2 \setminus L$ and satisfy (1.1) and (1.2) in this domain, the discontinuous solution of the equations of plane elasticity theory with the line L of jumps. If the quantities σ_{ij}, u_i are continuously continued on the boundary L almost everywhere (with the exception, perhaps, of a finite set of points), the boundary values of these quantities are locally summable functions on the contour L and the derivatives $\partial\sigma_{ij}/\partial x_k$, $\partial u_i/\partial x_k$ are locally summable functions in the space R^2, then we call such discontinuous solutions quasiregular.

The problem is to construct representations of discontinous solutions for a given jump line L and to indicate the conditions for which the discontinuous solution becomes quasiregular.

2. Systems of functional equations for discontinuous solutions. Let f^+ and f^- be boundary values to which the component f of a quasiregular discontinuous solution tends upon approaching the jump line L from the positive and negative normal side. Then the difference $f^+ - f^-$ determines the jump of this component on the contour L that we shall henceforth denote by $[f]$.

For an arbitrary quasiregular discontinuous solution the jumps $[\sigma_{ij}]$, $[u_i]$ considered as functions of the arc abscissa s are obviously locally summable on the contour L. At the same time the original quantities σ_{ij}, u_i and all their first derivatives are locally summable functions in R^2. Therefore, regular functionals from the space of generalized functions $D'(R^2)$ /2/, which will be related by the dependences

$$\partial\sigma_{ij}/\partial x_k = \partial_k\sigma_{ij} - [\sigma_{ij}]\cos\varphi_k\delta_L \tag{2.1}$$

$$\partial u_i/\partial x_k = \partial_k u_i - [u_i]\cos\varphi_k\delta_L \tag{2.2}$$

resulting from the properties of differentiation of generalized functions /3/, can be set in correspondence with the functions σ_{ij}, u_i, $\partial\sigma_{ij}/\partial x_k$, $\partial u_i/\partial x_k$. Here and henceforth, the same symbols are used to denote regular generalized functions as for the appropriate locally summable functions, ∂_k denotes the generalized derivative with respect to the coordinate x_k, and writing $m(s)\delta_L$ corresponds to a contour delta function with density $m(s)$ that is locally summable to the contour L. We later denote the convolution of two generalized functions by an asterisk $*$.

Eqs.(1.1) and (1.2) can be transformed in a natural manner into functional equalities for the appropriate regular generalized functions that will have the same form by virtue of the notation used.

Going over to the generalized derivative in (1.1) by using the dependence (2.1), we will have

$$\partial_j\sigma_{ij} = [p_i]\delta_L, \quad [p_i] = [\sigma_{ij}]\cos\varphi_j \tag{2.3}$$

where $[p_i]$ are jumps of the force vector components acting on the edge of the contour L.
Using relations (2.2) in Eqs.(1.2), similarly we obtain

$$\sigma_{ij} = \lambda(\partial_m u_m - [u_m]\cos\varphi_m\delta_L)\delta_{ij} + \mu(\partial_j u_i + \partial_i u_j - [u_i]\cos\varphi_j\delta_L - [u_j]\cos\varphi_i\delta_L) \tag{2.4}$$

Eqs.(2.3) and (2.4) are the complete system of equations to determine the functionals σ_{ij}, u_i in generalized derivatives.

Substituting (2.4) into (2.3), we arrive at the system of functional equations

$$\mu\partial_m\partial_m u_i + (\lambda+\mu)\partial_i\partial_j u_j = [p_i]\delta_L + \partial_k(\lambda[u_m]\cos\varphi_m\delta_L\delta_{ik} + \mu([u_i]\cos\varphi_k\delta_L + [u_k]\cos\varphi_i\delta_L)) \tag{2.5}$$

The set of generalized solutions of this system in $D'(R^2)$ obviously contains regular functionals corresponding to components of u_i of the original quasiregular discontinuous solution.

3. Construction of integral representations of discontinuous solutions.

Let us formulate the conditions which ensure the solvability of system (2.5) in the space $D'(R^2)$ by assuming that the jump functions $[p_i]$, $[u_i]$ on the right-hand sides of these equations are arbitrary locally summable functions given on a certain contour L.

Theorem 1. If the length of the contour L is finite, then generalized solutions of the system (2.5) exists in $D'(R^2)$ for arbitrary summable jump functions $[p_i]$, $[u_i]$ and can be represented in the form

$$u_i = u_i^\circ - U_{il} * r_l - \partial_k U_{il} * h_{lk} \tag{3.1}$$

$$r_l = [p_l]\delta_L \tag{3.2}$$

$$h_{lk} = \lambda[u_m]\cos\varphi_m\delta_L\delta_{lk} + \mu([u_l]\cos\varphi_k\delta_L + [u_k]\cos\varphi_l\delta_L)$$

where u_i° are solutions of the homogeneous system of equations in $D'(R^2)$

$$\mu\partial_m\partial_m u_i^\circ + (\lambda+\mu)\partial_i\partial_j u_j^\circ = 0 \tag{3.3}$$

and U_{il} are the matrix components of the fundamental solutions of the system under consideration that satisfy the equations

$$\mu\partial_m\partial_m U_{il} + (\lambda+\mu)\partial_i\partial_j U_{jl} = -\delta\delta_{il} \tag{3.4}$$

(δ is the delta-function from the space $D'(R^2)$ /2/).

Proof. Let Φ be the fundamental solution of the biharmonic operator

$$(\partial_i\partial_i)^2\Phi = \delta \tag{3.5}$$

which by the Malgrange-Ehrenpreis theorem /2/ always exists in $D'(R^2)$. Then explicit expressions for the matrix components of fundamental solutions of the system under investigation are given by the formulas

$$U_{ij} = \mu^{-1}(\lambda+2\mu)^{-1}((\lambda+\mu)\partial_i\partial_j\Phi - \delta_{ij}(\lambda+2\mu)\partial_l\partial_l\Phi) \tag{3.6}$$

that convert (3.4) into identities.

Taking into account that the supports of the functions $[p_i]\delta_L$ and $[u_i]\cos\varphi_j\delta_L$ agree with the set of points of the contour L that satisfies the compactness conditions in R^3, and taking into account the sufficient conditions for the existence of a convolution in $D'(R^2)$, we obtain that expressions (3.1) determine the generalized functions in $D'(R^2)$ for any set of functions $[p_i]$, $[u_i]$ summable on L.

Substituting representations (3.1) into the right-hand side of (2.5) and using the properties of a convolution of generalized functions /2/ and the relationships (3.2), (3.3), and (3.4), we obtain the chain of equalities

$$\begin{aligned}
&\mu\partial_m\partial_m u_i + (\lambda+\mu)\partial_i\partial_j u_j = \mu\partial_m\partial_m u_i^\circ + (\lambda+\mu)\partial_i\partial_j u_j^\circ - \\
&\quad(\mu\partial_m\partial_m U_{il} + (\lambda+\mu)\partial_i\partial_j U_{jl}) * r_l - \partial_k(\mu\partial_m\partial_m U_{il} + \\
&\quad(\lambda+\mu)\partial_i\partial_j U_{jl}) * h_{lk} = (\delta * r_l + \partial_k\delta * h_{lk})\delta_{il} = \\
&\quad r_i + \partial_k h_{ik} = [p_i]\delta_L + \partial_k(\lambda[u_m]\cos\varphi_m\delta_L\delta_{ik} + \\
&\quad\mu([u_i]\cos\varphi_k\delta_L + [u_k]\cos\varphi_i\delta_L))
\end{aligned}$$

from which it is seen that function of the form (3.1) are a generalized solution of system (2.5).

Using reductio ad absurdum, it can be shown that every generalized solution of the system under consideration has the form (3.1). The theorem is proved.

Corollary. A generalized solution of the system of functional Eq.(2.3) and (2.4) exists under the assumptions of Theorem 1 and is determined by the representations (3.1) for functionals

u_i and expressions for the functionals σ_{ij} obtained as a result of substituting the representation (3.1) into (2.4)

$$\begin{aligned}&\sigma_{ij} = \sigma_{ij}^{\circ} - L_{ijl} * r_l - \partial_k L_{ijl} * h_{lk} - h_{ij}\\&\sigma_{ij}^{c} = \lambda\partial_m u_m^{\circ}\delta_{ij} + \mu\,(\partial_j u_i^{\circ} + \partial_i u_j^{\circ})\\&L_{ijl} = \lambda\partial_m U_{ml}\delta_{ij} + \mu\,(\partial_j U_{il} + \partial_i U_{jl})\end{aligned} \tag{3.7}$$

Following /2/ we formulate the following lemma.

Lemma. Let $f(x), g(x) \in D'(R^2)$. If the support of the generalized function $g(x)$ ($\mathrm{supp}\, g(x)$) is a compact set in R^2 and the contraction of the generalized function $f(x)$ in $R^2 \setminus \{0\}$ belongs to the space of infinitely differentiable functions $C^\infty(R^2 \setminus \{0\})$ then the contraction of the convolution of these generalized functions in the domain $R^2 \setminus \mathrm{supp}\, g(x)$ belongs to $C^\infty(R^2 \setminus \mathrm{supp}\, g(x))$ and is expressed in terms of values of the functionals $g(x)$ in elements of the space $C^\infty(R^2 \setminus \{0\})$ by the formula

$$(f * g)(x) = (g(y), f(x-y)), \quad x \in R^2 \setminus \mathrm{supp}\, g, \quad y \in R^2 \tag{3.8}$$

If the solution of the biharmonic Eq.(3.5) is represented in the form (/4/, p.248)

$$\Phi = (8\pi)^{-1} |x|^2 \ln |x|$$

then the matrix components of the fundamental solutions found by means of (2.6) will be given by the expressions

$$\begin{aligned}&U_{ij} = \frac{1}{2\pi\mu(\varkappa+1)}\left\{\frac{x_i x_j}{|x|^2} - \delta_{ij}\left(\varkappa \ln|x| + \varkappa + \frac{1}{2}\right)\right\},\\&\varkappa = \frac{\lambda+3\mu}{\lambda+\mu}\end{aligned} \tag{3.9}$$

The contractions of these components in the domain $R^2 \setminus \{0\}$ are obviously analytic functions defined in this domain by the same relationships (3.9) in which, however, all the operations are treated as in the classical sense. It thence follows that the system of Eqs. (3.3) is of elliptic type and its homogeneous solutions are analytic functions everywhere in the space R^2 /2/.

By differentiating (3.9) in the generalized sense, analogous formulas can be obtained for the functionals $\partial_k U_{il}$, L_{ijl}, $\partial_k L_{ijl}$ and it can be seen that the contractions of these functionals in the domain $R^2 \setminus \{0\}$ are analytic functions.

Then on the basis of the lemma we arrive at the conclusion that contractions of the generalized functions u_i, σ_{ij} corresponding to the representations (3.1) and (3.7) in the domain $R^2 \setminus L$ determine infinitely differentiable functions whose explicit form is set up by using the equalities (3.8)

$$\begin{aligned}&u_i(x) = u_i^{c}(x) - (r_l(y),\ U_{il}(x-y)) - (h_{lk}(y),\ \partial_k U_{il}(x-y))\\&\sigma_{ij}(x) = \sigma_{ij}^{c}(x) - (r_l(y),\ L_{ijl}(x-y)) - (h_{lk}(y),\ \partial_k L_{ijl}(x-y))\\&x \in R^2 \setminus L,\ y \in R^2\end{aligned} \tag{3.10}$$

Taking into account the governing relationships for the contour delta-function /3/ (here and henceforth integration is over the contour L)

$$(m(s)\,\delta_L(y),\ \varphi(y)) = \int m(s)\varphi(t(s))\,ds$$

formulas (3.10) become

$$\begin{aligned}&u_i(x) = u_i^{\circ}(x) - \int \{[p_l](s) U_{il}(x - t(s)) +\\&\quad (\lambda\,[u_m](s)\cos\varphi_m(s)\delta_{lk} + \mu\,([u_l](s)\cos\varphi_k(s) +\\&\quad [u_k](s)\cos\varphi_l(s))\partial_k U_{il}(x - t(s))\}\,ds\\&\sigma_{ij}(x) = \sigma_{ij}^{\circ}(x) - \int \{[p_l](s) L_{ijl}(x - t(s)) +\\&\quad (\lambda\,[u_m](s)\cos\varphi_m(s)\delta_{lk} + \mu\,([u_l](s)\cos\varphi_k(s) +\\&\quad [u_k](s)\cos\varphi_l(s))\partial_k L_{ijl}(x - t(s))\}\,ds\end{aligned} \tag{3.11}$$

And since the jump functions $[p_l]$ and $[u_l]$ are, by definition, summable on the contour L, the functions in (3.11) will be analytic in the domain $R^2 \setminus L$.

Contracting the functional Eqs.(2.3) and (2.4) in the domain $R^2 \setminus L$ and taking the equality of the generalized and classical derivatives for the regular functionals u_i and σ_{ij} and the DuBois-Raymond lemma /2/ into account, it can be proved that analytic functions of the form (3.11) in $R^2 \setminus L$ satisfy the classical system of Eqs.(1.1) and (1.2).

We therefore arrive at the following theorem.

Theorem 2. On a contour L of finite length let there be given arbitrary summable functions $[p_l]$ and $[u_l]$. Then contractions of the functionals constructed by means of (3.1) and (3.7) in the domain R^2/L determine the discontinuous solution of the plane elasticity theory problem with the line of jumps L.

Remark. If it is assumed that the functions $[p_l]$, $[u_l]$ given on the contour L vanish at a certain point L_0 of this contour, then contractions of the functionals (3.1) and (3.7) in the domain $R^2 \setminus L_1$ ($L_1 = L \setminus L_0$) determine the discontinuous solutions of the equations of plane elasticity theory with the line of jumps L_1.

The proof of this assertion follows from the fact that the supports of the functions $[p_l]\,\delta_L$, $[u_l]\cos\varphi_k\delta_L$ coincide in this case with the set of points $L \setminus L_0$.

Therefore, the integral representations of discontinuous solutions for a finite contour L_1 consisting of separate open and closed piecewise-smooth arcs can be obtained from the integral representations for a continuous closed smooth contour L (possibly with points of selfintersection) formed by supplementing the contour L_1 by a set of smooth arcs L_0 if we set $[p_l] = [u_l] = 0$ on L_0 in the latter. This proves the possibility of limiting ourselves to a consideration of just some closed continuous smooth lines of a jump to analyse the integral representations constructed.

4. Conditions for the existence of quasiregular discontinuous solutions.

Let us now set up a set of sufficient conditions which enable a certain class of solutions of practical interest to be extracted from among the set of discontinuous solutions determined by (3.11). To this end, we extract the class of functions $H'(E)$ from the set of summable functions given on the contour L and having singularities at a finite set of points $E \subset L$.

We will say that the function $m(s)$ given on the contour L belongs to the class $H'(E)$ if it satisfies the Hölder condition on each of the sections L_ν of the contour L obtained as a result of its partition by a finite set of points E

$$| m(s_1) - m(s_2)| \leqslant A \mid t(s_1) - t(s_2)|^\gamma; \quad A, \gamma > 0$$
$$s_1, s_2 \in [a, b]: \ t(s_1), \ t(s_2) \in L_\nu$$

and in the neighbourhood of each point $t(s_\nu) \in E$ the following estimate holds:

$$m(s) = m_\nu^*(s) \mid t(s) - t(s_\nu)|^{-\alpha_\nu}, \quad 0 \leqslant \alpha_\nu < 1$$

where $m_\nu^*(s)$ is a function satisfying the Hölder condition in the neighbourhood of the point $t(s_\nu)$ /5/.

Theorem 3. Let L be a smooth continouous closed contour $(t(a) = t(b))$ on which a finite set of points $E \subset L$ is fixed, including all selfintersection points of the contour L, and summable functions $[p_l]$, $[u_l]$ are defined such that $[p_l] \in H'(E)$, $d[u_l]/ds \in H'(E)$ and for each point $t_\nu \in E$ of multiplicity n the following conditions are satisfied:

$$\sum_{p=1}^{n} \{[u_i](s_p + 0) - [u_i](s_p - 0)\} = 0 \tag{4.1}$$

where $\{s_p\}$ is a set of values of the arc abscissa corresponding to a given point $t_\nu \in E$. Then the discontinuous solution of the plane problem of elasticity theory with the jump line L determined by the relationships (3.11) is quasiregular.

Proof. We assume that summable functions $[p_l]$, $[u_l]$ satisfying all the conditions of Theorem 3 are given on the contour L. Then by Theorem 2, formulas (3.11) will determine the discontinuous solution of the plane elasticity theory problem with the jump line L. It remains to show that the boundary values of the components σ_{ij}, u_i of this solution exist for almost all points of the contour L and are summable functions of the arc abscissa s while the derivatives $\partial\sigma_{ij}/\partial x_k$, $\partial u_i/\partial x_k$ are locally summable functions in the space R^2.

Together with the closure of the contour the set of points E partitions the contour of integration L in the representations (3.11) into M smooth arcs that are not pairwise mutually intersecting. The set of values of the parameter s corresponding to the ends of these arcs will here consist of $M + 1$ elements $s_0 = a, s_1, \ldots, s_M = b$. Consequently, by partitioning the integration intervals in (3.11) in conformity with the proposed scheme and taking account of the initial assumptions as well as relationships (1.3), they can be represented in the form

$$\begin{aligned} u_i(x) &= u_i^\circ(x) - \alpha\mu^{-1}\Sigma \lim \{\langle [p_l](s)V_{il}(x - t(s))\rangle + \\ &\quad \langle \mu\,[u_l](s)(-dW_{il}(x - t(s))/ds)\rangle\} \\ \sigma_{ij}(x) &= \sigma_{ij}^\circ(x) - \alpha\Sigma \lim \{\langle [p_l](s)L_{ijl}(x - t(s))\rangle + \\ &\quad \langle \mu\,[u_l](s)(-dK_{ijl}(x - t(s))/ds\rangle\} \\ \alpha &= (2\pi(\varkappa + 1))^{-1}, \quad x \in R^2 \setminus L \end{aligned} \tag{4.2}$$

Here and henceforth, the angular brackets denote integration with respect to s between the limits $s_\nu + \varepsilon$ and $s_{\nu+1} - \varepsilon$, the limit is evaluated as $\varepsilon \to 0$, and summation is with respect to ν from $\nu = 0$ to $\nu = M - 1$. For the kernels $V_{il}(x)$, $W_{il}(x)$, $L_{ijl}(x)$, $K_{ijl}(x)$ we obtain expressions (in these and subsequent formulas $p, q = 1, 2$ and $p \neq q$, and summation is not carried out over p)

$$V_{pp}(x) = \varkappa \ln \frac{1}{r} + \frac{x_p^2}{r^2} - \varkappa - \frac{1}{2}, \quad V_{pq}(x) = \frac{x_1 x_2}{r^2} \tag{4.3}$$
$$W_{pp}(x) = (-1)^p \left(\frac{2x_1x_2}{r^2} - (\varkappa + 1)\operatorname{arctg}\frac{x_p}{x_q}\right),$$
$$W_{pq}(x) = (-1)^p \left(\frac{2x_q^2}{r^2} - (\varkappa - 1)\ln\frac{1}{r}\right)$$
$$L_{ppp}(x) = \left(\frac{4x_q^2}{r^2} - (\varkappa + 3)\right)\frac{x_p}{r^2},$$
$$L_{ppq}(x) = \left((\varkappa - 1) - \frac{4x_p^2}{r^2}\right)\frac{x_q}{r^2}$$
$$L_{12p}(x) = L_{21p}(x) = \left((1 - \varkappa) - \frac{4x_p^2}{r^2}\right)\frac{x_q}{r^2}$$
$$K_{ppp}(x) = (-1)^q \left(4 + \frac{8x_p^2}{r^2}\right)\frac{x_q}{r^2},$$
$$K_{ppq}(x) = (-1)^q \left(\frac{8x_q^2}{r^2} - 4\right)\frac{x_p}{r^2}$$
$$K_{12p}(x) = K_{21p}(x) = -K_{ppq}(x), \quad r = |x|$$

Since $d[u_l]/ds \in H'(E)$ and the kernels $W_{il}(x)$, $K_{ijl}(x)$ are analytic functions in the domain $R^2 \setminus \{0\}$ then for the second components under the summation sign in representations (4.2), integration by parts is applicable. Carrying out the operation mentioned in these formulas we obtain

$$u_i(x) = u_i^\circ(x) - \alpha \sum \Big\{\lim \Big\langle \mu^{-1}[p_l](s) V_{il}(x - t(s)) + \frac{d[u_l]}{ds} W_{il}(x - t(s))\Big\rangle - \lim [u_l](s_{\nu+1} - \varepsilon) W_{il}(x - t(s_{\nu+1})) + \lim [u_l](s_\nu + \varepsilon) W_{il}(x - t(s_\nu))\Big\}$$

and an analogous expression for $\sigma_{ij}(x)$. Grouping terms for identical values of the functions $W_{il}(x)$ and $K_{ijl}(x)$ in these last expressions and passing to the limit, taking conditions (4.1) into account, we will have

$$u_i(x) = u_i^\circ(x) - \alpha \int \Big\{\mu^{-1}[p_l](s) V_{il}(x - t(s)) + \frac{d[u_l]}{ds} W_{il}(x - t(s))\Big\} ds \tag{4.4}$$
$$\sigma_{ij}(x) = \sigma_{ij}^\circ(x) - \alpha \int \Big\{[p_l](s) L_{ijl}(x - t(s)) + \mu \frac{d[u_l]}{ds} K_{ijl}(x - t(s))\Big\} ds$$

Hence it follows that with the assumptions made regarding the jump functions $[p_l]$ and $[u_l]$, the representations (3.11) and (4.4) are equivalent.

We will now examine the singular integrals

$$I_p(x) = \frac{1}{\pi} \int \frac{m(s)(x_p - \xi_p(s))}{|x - t(s)|^2} ds,$$
$$J_p(x) = \frac{1}{\pi} \int \frac{m(s)\, 2(x_p - \xi_p(s))(x_q - \xi_q(s))^2}{|x - t(s)|^4} ds$$

On the basis of the Sokhotskii-Plemelj formulas (/5/, p.55) governing the boundary values of the Cauchy integral, analogous formulas can be obtained that govern the boundary conditions of these integrals for an arbitrarily given function $m(s)$ from the class $H'(E)$

$$I_p^\pm(t(s_0)) = \pm m(s_0)\cos\varphi_p(s_0) + \frac{1}{\pi}\int \frac{m(s)(\xi_p(s_0) - \xi_p(s))}{|t(s_0) - t(s)|^2} ds \tag{4.5}$$
$$J_p^\pm(t(s_0)) = \pm m(s_0)\cos\varphi_p(s_0)(\cos^2\varphi_p(s_0) - \cos^2\varphi_q(s_0)) + \frac{1}{\pi}\int \frac{m(s)\, 2(\xi_p(s_0) - \xi_p(s))(\xi_q(s_0) - \xi_q(s))^2}{|t(s_0) - t(s)|^4} ds$$

The integrals on the right-hand sides of these formulas should be considered in the principal value sense /5/.

We note that the boundary values of the integrals $I_p(x)$, $J_p(x)$ constructed for a certain $m(s) \in H'(E)$, do not exist for contour points belonging to the set E.

Starting from the representations (4.4) and (4.5) we find the boundary values of the quantities $u_i(x)$ and $\sigma_{ij}(x)$ for the jump line L

$$u_i^{\pm}(t_0) = u_i^{\circ}(t_0) \pm \frac{1}{2}[u_i](s_0) - \alpha \int \Big\{ \mu^{-1}[p_l](s) V_{il}(t_0 - t(s)) + \frac{d[u_l]}{ds} W_{il}(t_0 - t(s)) \Big\} ds \tag{4.6}$$

$$\sigma_{ij}^{\pm}(t_0) = \sigma_{ij}^{\circ}(t_0) \pm \frac{1}{2}(\varkappa + 1)^{-1} \tau_{ij}(s_0) - \alpha \int \Big\{ [p_l](s) L_{ijl}(t_0 - t(s)) + \mu \frac{d[u_l]}{ds} K_{ijl}(t_0 - t(s)) \Big\} ds,$$

$$t_0 = t(s_0)$$

$$\tau_{pp}(s) = [p_p]\cos\varphi_p(4\cos^2\varphi_q + \varkappa + 1) + [p_q]\cos\varphi_q(4\cos^2\varphi_q - \varkappa - 1) + (-1)^p 8\mu \Big\{ \frac{d[u_p]}{ds}\cos\varphi_q - \frac{d[u_q]}{ds}\cos\varphi_p \Big\} \cos^2\varphi_q$$

$$\tau_{pq}(s) = [p_1]\cos\varphi_2(\varkappa + 1 - 4\cos^2\varphi_1) + [p_2]\cos\varphi_1(\varkappa + 1 - 4\cos^2\varphi_2) + 8\mu \Big\{ \frac{d[u_1]}{ds}\cos\varphi_2 - \frac{d[u_2]}{ds}\cos\varphi_1 \Big\} \cos\varphi_1\cos\varphi_2$$

The formulas presented enable us to assert that the boundary values of the displacement and stress field components for a given discontinuous solution are summable functions on the contour L.

Finally, to prove that the functions $u_i, \sigma_{ij}, \partial u_i/\partial x_k, \partial\sigma_{ij}/\partial x_k$, obtained on the basis of the integral representations (4.4) are locally summable in the whose space R^2 it is sufficient to see that these function are summable in the whole space R^2, it is sufficient to see that these functions are summable in the whole circle of sufficiently small radius whose centre lies on the contour L. The proof is based on utilization of known estimates of the behaviour of a Cauchy-type integral and its derivative in the neighbourhood of the contour of integration and near singular points on this contour /5/.

This completes the proof of the theorem.

The remark expressed at the end of Sect.3 enables this theorem to be generalized to the case of discontinuous solutions with an arbitrary piecewise-smooth jump line.

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